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SHORTER COMMUNICATIONS

PROPERTIES OF SPECULARLY REFLECTED RADIATION AT NORMAL INCIDENCE

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MEASUREMENT of the absolute specular reflectance at normal incidence is difficult owing to the physical interference between the radiation source and the detector. It is, therefore appropriate to deduce relevant properties of the normal specular reflectance by analytical means. In this note, consideration is given to systems consisting of one or more absorbing films on an absorbing substrate, as is pictured schematically in Fig. 1. In the figure, the films are denoted by 1, 2, ... (film thicknesses h_1, h_2, ...), the nonparticipating environment by e, and the substrate by s. The angle of incidence is θ_i.

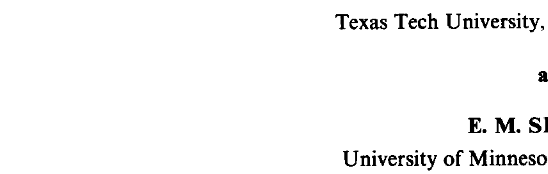

FIG. 1. Film–substrate systems.

It is to be demonstrated that for the monochromatic specular reflectance R and the corresponding phase change δ_r, the derivatives $\partial R/\partial\theta_i$ and $\partial\delta_r/\partial\theta_i$ are zero at $\theta_i = 0°$. These findings are shown to be valid for either single films or multiple films, and for either perpendicular or parallel polarized radiation. In arriving at these results, it is also shown that at the interface between adjacent media, the derivatives of the interfacial reflection coefficients and interfacial phase changes are zero at normal incidence, regardless of whether the media adjoining the interface are absorbing or nonabsorbing. As a consequence of these properties, measurements at small incidence angles θ_i can be extrapolated to provide information at $\theta_i = 0°$.

SINGLE FILMS

The monochromatic specular reflection coefficient r for the film-substrate system shown in the left-hand diagram of Fig. 1 follows by generalizing a result given by Born and Wolf [1], so that

$$r = \rho e^{i\delta_r} = \frac{\rho_{e1}\, e^{i\phi_{e1}} + \rho_{1s}\, e^{-2v_1\eta_1}\, e^{i(\phi_{1s} + 2u_1\eta_1)}}{1 + \rho_{e1}\,\rho_{1s}\, e^{-2v_1\eta_1}\, e^{i(\phi_{e1} + \phi_{1s} + 2u_1\eta_1)}} \tag{1}$$

which applies for optically smooth surfaces; homogeneous, isotropic, non-scattering media; and wavelengths greater than those at which quantum effects are important. In equation (1), ρ_{e1} and ρ_{1s} are the amplitudes of the interfacial reflection coefficients, and ϕ_{e1} and ϕ_{1s} are the corresponding phase shifts. In addition, $\eta_1 = (2\pi/\lambda)h_1$, where λ is the wavelength of the radiation in vacuum and h_1 the film thickness. The optical constants of the media are expressed via the complex indices of refraction as

$$\hat{n}_e = n_e, \qquad \hat{n}_1 = n_1(1 + i\kappa_1), \qquad \hat{n}_s = n_s(1 + i\kappa_s) \tag{2}$$

and the quantities u and v are defined by the relations

$$\hat{n}_1 \cos\theta_1 = u_1 + iv_1, \qquad \hat{n}_s \cos\theta_s = u_s + iv_s \tag{3}$$

where the angles θ_i, θ_1, and θ_s are related by Snell's law: $n_e \sin\theta_i = \hat{n}_1 \sin\theta_1 = \hat{n}_s \sin\theta_s$.

Inasmuch as the reflectance $R = |r|^2$, equation (1) can be manipulated to yield

$$R = \frac{\rho_{e1}^2\, e^{2v_1\eta_1} + \rho_{1s}^2\, e^{-2v_1\eta_1} + 2\rho_{e1}\,\rho_{1s}\cos(\phi_{1s} - \phi_{e1} + 2u_1\eta_1)}{e^{2v_1\eta_1} + \rho_{e1}^2\,\rho_{1s}^2\, e^{-2v_1\eta_1} + 2\rho_{e1}\,\rho_{1s}\cos(\phi_{1s} + \phi_{e1} + 2u_1\eta_1)} \tag{4}$$

$$\tan\delta_r = \frac{\rho_{1s}(1-\rho_{e1}^2)\sin(2u_1\eta_1+\phi_{1s})+\rho_{e1}(e^{2v_1\eta_1}-\rho_{1s}^2 e^{-2v_1\eta_1})\sin\phi_{e1}}{\rho_{1s}(1+\rho_{e1}^2)\cos(2u_1\eta_1+\phi_{1s})+\rho_{e1}(e^{2v_1\eta_1}+\rho_{1s}^2 e^{-2v_1\eta_1})\cos\phi_{e1}}. \tag{5}$$

Consider now the derivative $\partial R/\partial\theta_i$. As a shorthand, we write $R = (A + B + C)/(D + E + F)$, so that

$$\frac{\partial R}{\partial\theta_i} = \left[\frac{\partial A}{\partial\theta_i}+\frac{\partial B}{\partial\theta_i}+\frac{\partial C}{\partial\theta_i} - R\left(\frac{\partial D}{\partial\theta_i}+\frac{\partial E}{\partial\theta_i}+\frac{\partial F}{\partial\theta_i}\right)\right]\Big/(D+E+F). \tag{6}$$

It may be verfied that $\partial A/\partial\theta_i, \ldots, \partial F/\partial\theta_i$ involve $\partial u_1/\partial\theta_i$, $\partial v_1/\partial\theta_i$, $\partial\rho_{e1}/\partial\theta_i$, $\partial\rho_{1s}/\partial\theta_i$, $\partial\phi_{e1}/\partial\theta_i$, and $\partial\phi_{1s}/\partial\theta_i$. Since

$$2u_1^2 = a_1 + (a_1^2+b_1)^{\frac{1}{2}}, \qquad 2v_1^2 = -a_1 + (a_1^2+b_1)^{\frac{1}{2}} \tag{7}$$

where

$$a_1 = n_1^2(1-\kappa_1^2) - n_e^2\sin^2\theta_i, \qquad b_1 = 4n_1^4\kappa_1^2 \tag{7a}$$

it follows that $\partial u_1/\partial\theta_i$ and $\partial v_1/\partial\theta_i$ are each proportional to $\sin\theta_i$, so that

$$\partial u_1/\partial\theta_i = \partial v_1/\partial\theta_i = 0 \text{ at } \theta_i = 0. \tag{8}$$

The forms of the derivatives $\partial\rho_{e1}/\partial\theta_i$, $\partial\rho_{1s}/\partial\theta_i$, $\partial\phi_{e1}/\partial\theta_i$, and $\partial\phi_{1s}/\partial\theta_i$ depend on the polarization of the radiation. For perpendicular-polarized radiation

$$\rho_{e1}^2 = \frac{(n_e\cos\theta_i - u_1)^2 + v_1^2}{(n_e\cos\theta_i + u_1)^2 + v_1^2},$$

$$\tan\phi_{e1} = \frac{2v_1 n_e\cos\theta_i}{u_1^2+v_1^2-n_e^2\cos^2\theta_i} \tag{9}$$

$$\rho_{1s}^2 = \frac{(u_1-u_s)^2+(v_1-v_s)^2}{(u_1+u_s)^2+(v_1+v_s)^2},$$

$$\tan\phi_{1s} = \frac{2(u_s v_1 - u_1 v_s)}{u_1^2-u_s^2+v_1^2-v_s^2} \tag{10}$$

in which u_s and v_s are expressed by equations (7) and (7a) with subscript 1 everywhere replaced by subscript s. It can be verified that $\partial\rho_{e1}/\partial\theta_i$, $\partial\rho_{1s}/\partial\theta_i$, $\partial\phi_{e1}/\partial\theta_i$, and $\partial\phi_{1s}/\partial\theta_i$ are proportional to $\sin\theta_i$, and so

$$\partial\rho_{e1}/\partial\theta_i = \partial\rho_{1s}/\partial\theta_i = \partial\phi_{e1}/\partial\theta_i = \partial\phi_{1s}/\partial\theta_i = 0 \text{ at } \theta_i = 0. \tag{11}$$

In view of equations (8) and (11), it follows that for perpendicular-polarized radiation, the derivatives $\partial A/\partial\theta_i, \ldots, \partial F/\partial\theta_i$ that appear in equation (6) are zero at $\theta_i = 0°$. A similar result is obtained for parallel-polarized radiation. Thus, from equation (6)

$$\partial R/\partial\theta_i = 0 \text{ at } \theta_i = 0. \tag{12}$$

By proceeding along similar lines and employing equation (5), it can be shown that

$$\partial\delta_r/\partial\theta_i = 0 \text{ at } \theta_i = 0. \tag{13}$$

Equations (11)–(13) express the desired results for film-substrate systems having a single film.

MULTIPLE FILMS

To demonstrate the results for multiple films, it is sufficient to show how to deal with two films. The notation for the two-film case is indicated in the right-hand diagram of Fig. 1. The reflection coefficient r for such a film–substrate system may be written as

$$r = \rho e^{i\delta_r} = \frac{\rho_{e2}e^{i\phi_{e2}} + \rho^* e^{-2v_2\eta_2}e^{i(\phi^*+2u_2\eta_2)}}{1+\rho_{e2}\rho^* e^{-2v_2\eta_2}e^{-i(\phi_{e2}+\phi^*+2u_2\eta_2)}}. \tag{14}$$

The quantities ρ^* and ϕ^* are the amplitude and phase shift of a film–substrate system consisting of film 1 and substrate s. By following steps similar to those in the derivation of equations (4) and (5), the reflectance R and the phase shift δ_r for the two-film case can be derived as

$$R = \frac{\rho_{e2}^2 e^{2v_2\eta_2} + (\rho^*)^2 e^{-2v_2\eta_2} + 2\rho_{e2}\rho^*\cos(\phi^*-\phi_{e2}+2u_2\eta_2)}{e^{2v_2\eta_2} + \rho_{e2}^2(\rho^*)^2 e^{-2v_2\eta_2} + 2\rho_{e2}\rho^*\cos(\phi^*+\phi_{e2}+2u_2\eta_2)} \tag{15}$$

$$\tan\delta_r = \frac{\rho^*(1-\rho_{e2}^2)\sin(2u_2\eta_2+\phi^*)+\rho_{e2}[e^{2v_2\eta_2}-(\rho^*)^2e^{-2v_2\eta_2}]\sin\phi_{e2}}{\rho^*(1+\rho_{e2}^2)\cos(2u_2\eta_2+\phi^*)+\rho_{e2}[e^{2v_2\eta_2}+(\rho^*)^2e^{-2v_2\eta_2}]\cos\phi_{e2}}. \tag{16}$$

In the foregoing, $\eta_2 = (2\pi/\lambda)h_2$, and u_2 and v_2 are given by equations (7) and (7a) in which subscript 1 is replaced by subscript 2.

Since the system consisting of 1 and s corresponds closely to that analyzed in the preceding portion of the paper, it follows that

$$\partial\rho^*/\partial\theta_i = \partial\phi^*/\partial\theta_i = 0 \text{ at } \theta_i = 0. \tag{17}$$

For concreteness, consideration may be given to perpendicular polarized radiation; correspondingly, the ρ_{e2} and ϕ_{e2} that appear in equations (15) and (16) are expressed by equations (9) with subscript 1 replaced by subscript 2.

The derivative $\partial R/\partial\theta_i$ at $\theta_i = 0$ may now be examined. Since $\partial\rho_{e2}/\partial\theta_i$, $\partial\phi_{e2}/\partial\theta_i$, $\partial u_2/\partial\theta_i$, and $\partial v_2/\partial\theta_i$ are all proportional to $\sin\theta_i$, and $\partial\rho^*/\partial\theta_i$ and $\partial\phi^*/\partial\theta_i$ are zero, it follows that $\partial R/\partial\theta_i = 0$ at $\theta_i = 0$. It can be shown that this property also holds for parallel polarized radiation and also that $\partial\delta_r/\partial\theta_i = 0$.

If there are three films, then the vanishing of $\partial R/\partial\theta_i$ and $\partial\delta_r/\partial\theta_i$ at $\theta_i = 0$ can be demonstrated by lumping together the substrate and the two lowermost films and employing the findings of the prior paragraphs; and so on for any number of films.

CONCLUDING REMARKS

Typical experimental reflectance data, taken from [2], are presented in Fig. 2. The data are for perpendicular polarized radiation at a wavelength of 0·633 μ. The upper and lower sets of data correspond respectively to an aluminum oxide film ($h = 1{\cdot}71$ μ) on an aluminum substrate and to a zirconium oxide film ($h = 1{\cdot}51$ μ) on an aluminum substrate. The solid lines represent the predicted reflectance versus angle distribution as evaluated from equation (4). Examination of the figure reveals that accurate reflectance information at $\theta_i = 0°$ can be obtained by using the properties derived here as a guide for extrapolating the data.

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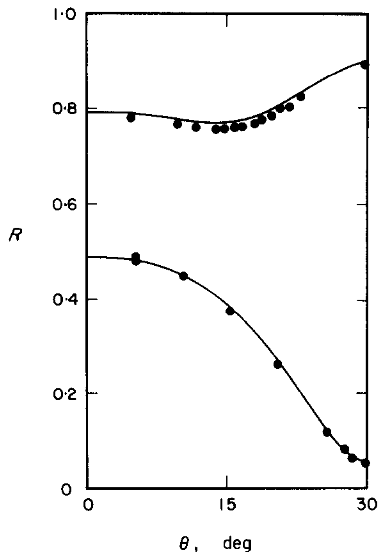

Fig. 2. Typical reflectance data

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HEAT TRANSFER TO STEAM FLOWING TURBULENTLY IN A PIPE

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NOMENCLATURE

a, constant equal to 1·0;
A, total band absorptance;
b, constant equal to 1·25;
B, radiation intensity;
c_p, specific heat at constant pressure;
k, thermal conductivity;
Nu, $= q_0 2r_0/k(T_0 - T_b)$, Nusselt number;
q, heat flux;
r, radial coordinate;
Re, $= u_b 2r_0/\nu$ Reynolds number;
T, temperature;
u, velocity.

Greek symbols

β, $= q_0(\tau_0/\rho_0)^{\frac{1}{2}}/c_{p_0}\tau_o T_o$, heat-transfer parameter;
γ, angle;
μ, dynamic viscosity;
ν, $= \mu/\rho$ kinematic viscosity;
ρ, density;
τ, shear stress;
ω, wave number.

Subscripts

b, bulk value;
c, value at band center;
o, evaluated at wall.